The Finite, the Infinite, and God

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In a set theory class in graduate school we got to discussing Cantor theory. This theory discusses the relative sizes of different infinities. A nun in the class said to me “God is infinity and infinity is God”. A basic cliché we all use is that God is infinite. Naturally her statement was an over-simplification. The comment sparked an interest, however, that has deepened with time.

Ellen White speaks not only of the infinite value of God, but of sin when she writes: “The infinite value of the sacrifice required for our redemption reveals the fact that sin is a tremendous evil.” [16, p451] Elsewhere she comments that our finite minds cannot hope to comprehend all of the infinity that is God.

The purpose of this paper is to explore a little of what infinity is and how it can influence our understanding of our Creator and our relation to him. Tantalizing pieces can be shared with the students when an appropriate topic arises. Hopefully this will start some of our students thinking.

An incident will indicate that some have considered this topic of paramount importance. Giordano Bruno, amongst others, taught that space was infinite. He strongly advocated this concept in his dialogue of 1584 “On the Infinite Universe and Worlds” For this he was lured to Venice, captured and turned over to the inquisition. After 9 years of imprisonment and torture, during which time he refused to recant, he was burned at the stake. [11, p19]

Infinity, as used by Christians, is most often applied as an attribute of God. It is one of those omni- words: omnipotent, omnipresent, and omniscient. It gives visions of eternity, past and present. Always it is the distantly unreachable and mysterious.

Harry Blamires encourages us to explore the unknown.

There is nothing in our experience, however trivial, worldly, or even evil, which cannot be thought about christianly. To think christianly is to accept all things with the mind as related, directly or indirectly, to man’s eternal destiny as the redeemed and chosen child of God. [3, p44, 45]
Almost fifty years ago George McCready Price urged us to apply modern discoveries to the understanding the character of the Creator.

All these modern discoveries tend to present the problems of the universe in aspects never dreamed of by the people of the Victorian age. Yet we see little inclination on the part of anyone to make serious use of these new discoveries in trying to understand the character of the Creator, or in attempting to study the ways in which He is conducting His universe. [10, p17]

Scripture

A study of the context of for ever, eternal, everlasting, infinity, for evermore, and innumerable, in a concordance of the King James Version of the Bible is essential before we go too deep into the mathematical context of infinity.

In Scripture, these words often have a relative meaning. By this I mean that there may be an expiration. On the other hand we often take them to mean absolutely what they say. Some examples of relative use, in a traditional Adventist context are these:

Job is accused of having committed an infinite number of sins [2, Job 22: 5] The armed strength of Ethiopia and Egypt is listed as infinite, and yet they were carried into captivity. [2, Nahum 3:9] There are innumerable beasts, small and great in the sea, and innumerable grains of sand by the sea shore. [2, Psalm 104: 25; Hebrews 11: 12.]

Jude 7 talks of an eternal fire that consumed Sodom and Gomorrah. Leviticus lists statutes for ever that pertained to the temple ceremonies and passed away with the old dispensation. [see 2, Leviticus 6: 18; 7: 34, 36; 10: 9, 15; etc.] The Aaronic priesthood, termed an everlasting priesthood in Exodus, is replaced by a more excellent ministry in Hebrews. [2, Exodus 40: 15; Hebrews 8: 6.]

On the other hand these same words are used in a context that we take in an absolute sense. God is an everlasting king. [2, Jeremiah 10: 10.] His understanding is infinite. [2, Psalm 147: 5.] And who doubts for a moment that the promise of eternal life in John 3: 16 is not really for ever? If that is not true, then indeed we are of all men most miserable.
From these and other texts it would appear that eternity or infinity is something that lasts a long
time or is extremely numerous, and, most importantly, is beyond the power or authority of man to
meddle with.

Other Early Views of Infinity

The ancient Greeks used the word apeiron (ἄπειρον) for infinity. It appeared to have many
negative connotations and none positive. There was no place for the infinite in the world of Plato
or Pythagoras. The Pythagoreans believed everything could be described by a finite arrangement
of the natural numbers. Aristotle recognized that there were some aspects of the world that might
need apeiron. Things such as the flow of time, and the number of points on a line segment might
indeed be infinite. He invented the notions of potentially infinite versus actually infinite.
Something actually infinite had an air of finality and completeness to it. The set of natural numbers
{1, 2, 3, ...} on the other hand, was only potentially infinite because one never used or needed
more than a finite number of them at any one time. Thus he could remain in his own comfortable
little finite world without dealing with the unpleasant concept of infinity.

St. Augustine, who adapted Platonic philosophy to Christianity, believed that both God and his
thoughts were infinite. This was a bold step beyond believing simply that God was nebulously
infinite. Augustine's successors would not go as far as he did. St. Thomas Aquinas gives a
circular "proof" that it would be impossible for God to create anything infinite. (A circular proof
assumes the result as part of the hypotheses.)

Infinity

George Bernard Shaw once astutely observed that to a bushman who can only count as far as
his ten fingers, to him "eleven is an incalculable mystery." [5, p42] Of course Shaw was
underestimating even the most primitive of the human race. But his point is well taken that infinity
is merely something quantitatively beyond our reckoning.
Galileo Galilei in the early 1600's was the first person to begin to approach infinity in a modern spirit. He had learned something from Bruno's experience and so was more careful in how he expressed his scientific sentiments. History documents that he was not careful enough however.

Galileo supposed that there were as many points on a 3 inch line as on a 6 inch line. He argued that there were as many points on the circumference of a circle of radius two inches as there were on a circle of radius one inch. To see this, draw the two circles with a common center.

Then for every point B on the larger circle there exists a corresponding point A on the smaller circle where the radius from the original point meets the smaller circle. In this fashion every point on the larger circle could be paired with one on the smaller circle. So there must be an equal number of points on each circle, even though the circumference of the larger was twice as long as that of the smaller. This type of reasoning was not fully utilized until Georg Cantor wrote his set theory 250 years later.

Georg Ferdinand Ludwig Philipp Cantor (1845-1918), intrigued by recent non-Euclidean geometry theory, created classical set theory, to put geometry back on track as a respectable branch of mathematics. As part of his set theory he faced the issue of how to count the number of elements in a set. He arrived at the fruitful technique of comparing two sets, element by element. If he ran out of elements in one of the sets first, then that set had fewer elements.

Cantor opened his remarks with the following bold statement:

The fear of infinity is a form of myopia that destroys the possibility of seeing the actual infinite, even though it in its highest form has created and sustains us, and in its secondary transfinite forms occurs all around us and even inhabits our minds. [11, p43]

He recognized an absolute infinity. Between this absolute infinity and the finite numbers he created a new class of numbers he called the transfinite numbers.

He denoted the number of integers as \( \omega \). This is the smallest infinity. Infinite sets have the quality that their elements can be paired with those of proper subsets of themselves. This can, of
course, violate our intuition. For example, Galileo used the following pairing to show that there are as many numbers as perfect squares (a proper subset of the set of positive integers):

\[
\begin{array}{ccccccc}
1 & 2 & 3 & 4 & 5 & 6 \\
\uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \ldots \\
1 & 4 & 9 & 16 & 25 & 36
\end{array}
\]

Cantor used the same argument to show that there are no more fractions than natural numbers. However the number of the points on a line segment is greater than \(\omega\). Thus there are many different infinities each one as much bigger than the previous as \(\omega\) is bigger than any finite number.

David Hilbert used the first letter of the Hebrew alphabet to denote infinity. Thus \(\aleph_0\) had the same value as Cantor’s \(\omega\). Continuing from there, the set of all subsets of the natural numbers has more elements than the set of natural numbers, so he denoted that infinity as \(\aleph_1\). The set consisting of the set of all subsets of the set of all subsets of the natural numbers would be denoted by \(\aleph_2\). Furthermore this process can obviously be continued indefinitely.

We can now settle the medieval controversy over how many angels can stand on the head of a pin. If each angel requires but a single point to stand on and there are \(\aleph_0\) angels, the head of the pin could contain all of the angels in the universe. There would still be infinitely more than \(\aleph_0\) points remaining unoccupied!

**God is Infinite**

This view of an absolute infinity far greater than the ordinary infinities we meet with every day gives us a new and exalted view of what infinity can mean. The eternal life we all look forward to consists of \(\omega\) or \(\aleph_0\) years, but the God with whom we’ll spend it represents the only thing we can conceive of as being an absolute infinity.

The famous mathematician and philosopher Bertrand Russell points out that the infinity of rationals \([\omega]\) does not demand, for its definition or use, any infinite classes or infinite integers. It is not, in actual fact a very important notion, and we could dispense with it altogether if there were any object in doing so. The Cantorian \([\text{absolute}]\) infinite, on the other hand, is of greatest and most fundamental importance; the understanding of it opens the way to whole new realms of mathematics and philosophy. [12, p65]
In the context of Cantor's absolute infinity we quote St. Gregory: "'No matter how far our mind may have progressed in the contemplation of God, it does not attain to what he is but to what is beneath him.'" [5, p48]

In his world view James Sire defines a God who is omniscient:

This means that God is all-knowing. He is the alpha and the omega and knows the beginning from the end. He is the ultimate source of all knowledge and all intelligence. He is He Who Knows. The author of Psalm 139 expresses beautifully his amazement at God's being everywhere, preempts him—knowing him even as he was being formed in his mother's womb! [13, p27]

Sire thus believes that God's knowledge is infinite in at least two dimensions. God knows everything and he knows all time.

In the balance of the paper, I wish to explore some of the ideas that I have had. Many of these are not original, of course, but many have occurred to me independently of sources I discovered later. They should all be read with the following caveat: Our thinking in this realm is probably as rudimentary as that of the Greek philosophers 2500 years ago, who believed that all heavenly motion must occur in perfect circles. It will be very surprising if, in the earth made new, any one or another of these specific theories described below are found to be true. [Adapted with apologies to one of this country's greatest astronomers, the late George O. Abell; 1, p378.] "Eye hath not seen, nor ear heard neither have entered into the heart of man, the things which God hath prepared for them that love him!" [2, I Corinthians 2:9]

**Bounded Infinity**

The term bounded infinity seems at first sight to be an oxymoron. It refers to the phenomenon mentioned earlier and stated originally by Galileo that a line segment (which is bounded) contains as many points as any other line segment of any length (bounded or unbounded). Cantor showed that a line segment consists of an uncountably infinite number of points. Uncountably infinite means a transfinite number greater than Cantor's $\omega$.

Mandelbrot points out that the common land boundary between Spain and Portugal is listed as 987 km versus 1214 km in their respective encyclopedias. [8, p 33] He attributes this difference to
their each using a measuring device with different resolution. In fact the shorter the measuring rod, the longer the boundary. The reason is that the shorter rod will measure more of the irregularities in the boundary. Taken to its logical conclusion, as the length of the measuring rod gets closer and closer to zero, the measured length of the boundary will get larger and larger until it approaches \( \infty \). Mandelbrot proceeds to define a \textit{fractal dimension} (or fractional dimension). We will discuss dimension later on.

Nature tends to abound in curves that yield infinite lengths when measured with infinitesimal measuring rods. Until recently, mathematics has not concerned itself with irregular curves such as these. In fact it still does not have a method of modelling such curves. They are too complicated.

Scientists have defined the chemical structure of matter in terms of molecules. These in turn are described in terms of atoms, which were originally considered indivisible. Atoms are now considered to consist of particles such as protons, electrons, and neutrons. Recently some of these particles have been subdivided into still smaller subparticles called quarks. And surely the end is not in sight. Will matter turn out to be infinitely divisible?

Behind all of this infinite variety in the smallness of nature, we see a Creator. He has a design for nature that goes smaller and deeper than our most sensitive instruments.

Ponder for a moment the mystery of the electron. It’s electro-magnetic force is subject to the well known inverse square law. That means that the force quadruples as the distance to the electron is halved. Since the electron is practically dimensionless, an object “touching” the electron would experience an unlimited (infinite) force. Here is an infinite force bound up in an infinitesimal point.

The computer age yields two more examples. Technology has been squeezing more and more memory into the same physical dimensions every year. In fact, in recent years, they have doubled the amount of memory on an integrated circuit every year. Yet computer memory has not approached that contained within the human mind in either capacity or ease of accessibility. God has enabled our brains to remember an apparently unlimited amount of knowledge. Although the
human body and mind have degenerated considerably since the perfection of creation, we are still blessed with far more memory capacity than we ever use.

But God knows the name of every star [2, Psalm 147: 4; Isaiah 40:26]. Scientists tell us that there are probably at least 300 billion stars in our galaxy alone. [14, p 422] To get a slight feeling for how big that number is, suppose we gave each star a word out of the Oxford English Dictionary for a name. We would run out of names at about 500 thousand stars. How many of us know even 20% of that number of English words? The United Nations estimates that there are about 2000 different languages in the world. If every language had as rich a vocabulary as English (and most don’t) and we used dictionaries of each of these languages to names the stars in our Milky Way galaxy we could name about 1 billion stars. That is less than one per cent of the stars in our galaxy alone. Furthermore there are over 200 billion known galaxies of stars. Now marvel at the memory that knows all of that and at the same time knows exactly how many hairs you have on your head! Are we viewing a memory that just might be infinite? I believe we are.

When people think of computers, they normally think of speed. Today we measure the speed of computers in MIPS (millions of instructions per second). For example the Macintosh computer I am typing this paper on is rated at about 8 MIPS. My old Apple II+ was rated at about 1 MIPS. Newer and faster computers run at much higher MIPS rates. What we are doing is squeezing more and more processing into each second. I believe that God must have the ability to do an infinite amount of processing in any given time period. The amount of data handled needs to be measured in the realm of $\aleph_0$ at least and maybe some higher $\aleph_n$ is required. Like some of the new modern computers and like the human brain, God’s mind must be able to process many things in parallel (simultaneously). Again I believe that he must be able to handle at least $\aleph_0$ unrelated thoughts simultaneously.

**Unbounded Infinity of Time**

As the doctrine of the eternal existence of God began to dawn on my childhood mind I was utterly intrigued. Everything in our experience has a beginning. And so my question was “What
happened before that?" People suggested I think about a ring, because it has long been a symbol of infinity, having neither beginning nor end. But, that didn't help a bit.

St. Augustine asked himself what God was doing before he made heaven and earth, to which he replied "He was creating hell for people who ask questions like that!" [7, p112].

Looking out at the nighttime heavens we see objects that are thousands of light years distant. This implies that these objects existed long before the 6,000 year old beginning of the heavens and earth. A simple minded explanation is that when God made the stars he made them with apparent age, that means that he made their light beams throughout the universe at the same time he made them. After all, didn't he make Adam and Eve and their garden home fully mature, with every evidence of age?

Even our simple minded answer, however, leaves us with St. Augustine's nagging question, what was God doing before then? In *God and the Astronomers* Robert Jastrow traces the tortuous trail of modern cosmology. He shows how the galactic red shift forced Astronomers to draw an obvious conclusion. The apparent interpretation of the red shift is that the universe is expanding. Now simply trace this expansion backwards to the time when all these lines of expansion converge and you come up with a date. This date, 10 to 20 billion years ago, would seem an obvious choice for the beginning of time in our universe. The event is referred to as the Big Bang. There are two straightforward methods of arriving at this date. One points to about 20 billion years ago, the other to about 10. But the allowance for error in each approximation overlaps the other, so no one is worried about the discrepancies, yet. This forced scientists to accept that the universe did have a beginning, it has not always been here. Jastrow states, albeit reluctantly, that the modern scientist has

scaled the mountains of ignorance; he is about to conquer the highest peak; as he pulls himself over the final rock, he is greeted by a band of theologians who have been sitting there for centuries! [7, p116]

Naturally he places the creation of the universe at the time of the Big Bang. Of course neither creationists nor evolutionists believe the earth was created at that time.
It seems reasonable to believe that God created our universe all within a finite span of time. This would tend to imply that he has had an infinite amount of time, if indeed he is subject to time, to create other universes. More on this later.

Throughout the entire “life” of the universe, no matter how long or short you make it, the universe has been changing. Were Abraham alive today he would notice that Arcturus has visibly moved from its position amongst the other stars in the vicinity. He would notice, too, that our North Star, is not what he knew as the north celestial pole in his day.

Holmes credits the concept that good means immutable and evil means change to the effect of Platonism on Christian thought. [6, p73] Apparently God is the only thing that is “the same yesterday and to day and for ever.” [2, Hebrews 13:8]

Were this earth to last long enough, some great changes would take place. The most major is that the sun would eventually run out of fuel. This would end life as we know it on this earth. But surely that is not the destiny planned for sinless beings by a loving God!

When God made man in his image and told him to be fruitful and multiply and fill the earth, did he mean that man was to stop procreating upon filling the earth? Maybe! But I doubt it. Far more likely, in my opinion, God originally meant for the human race to continue growing and multiplying until it filled the universe not just this tiny speck of dust in his vast creation. If one is to believe Carl Sagan in his TV mini series Cosmos, at least a tenth of the stars we see in the night sky must have planets about them. Many of these are possibly inhabitable. Very possibly they are just Garden of Edens waiting for us to take up residence.

Mathematically this expanding human race would mean that at any time in the future the number of humans would still be finite. However over the vast eons of eternity this number would grow bigger than any finite number you might name. Thus our universe would need at least \( \aleph_0 \) planets eventually. Very possibly God might continue to create more as the need arose. When our present sun did exhaust all of its fuel, he could either give it a new transfusion of energy, or encourage those remaining on this planet to move on.
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Unbounded Infinity of Space.

The majority of scientists today believe that our universe is extremely large, but finite. If it is indeed true that it is finite, then God created the whole universe in a finite amount of time. In other words, if the universe is finite there must be a first thing that was created. Furthermore it can only occupy a finite amount of space. In other words there must be something that is the farthest away from us. If we take the distance to the farthest object from us, we can use this distance to create a sphere with that radius about us that contains the entire universe. What is beyond this vast sphere? No matter how great the sphere, there is still an infinity beyond.

It seems reasonable that there are more universes beyond ours. In fact it would be highly unreasonable to imagine that a God who is eternal would create exactly one finite universe. Otherwise he would have had an eternity to contemplate his one “tiny” creation!

The majority of these scientists who think the universe is finite, also think that space is curved. Think of the curvature in this fashion. In the days of Columbus, most people believed the surface of the earth to be flat. This meant that if you sailed out in any direction you would eventually fall off the edge of the earth. Now we have conclusive evidence (he and others sailed far enough) to know that the surface of the earth is curved, so that if we traveled in any direction long enough we would arrive back at our starting position. In order for this surface to form a sphere, it has to be curved in a direction not in the plane of the surface. Thus curvature of the two dimensional surface of the earth requires a third spatial dimension, undetectable by the compass.

It is just as possible that if we traveled by super fast spacecraft in any single direction far enough we would find ourselves back exactly were we started. In order for this to be possible our three dimensional space must be embedded in a higher dimensional space, a fourth dimension.

Many of these concepts of the nature of the universe began as I studied higher dimensions in mathematics. They began to gel into a model as a result of a discussion I had with Dr de Groot, the senior astronomer at the Armagh Observatory in Ireland in the late 1970’s. He had come to Cape Town for an international astronomical conference. He had been a Seventh-day Adventist
for a number of years and had been struggling with reconciling the find­
ings of science with Adventist views of creation.

Dr. de Groot’s universe can be represented by the following dia-
gram. Each circle in the diagram represents a three dimensional uni-
verse. Ours is but one of these circles (universes). There may in-
deed be an infinite number of these universes. Heaven, God’s throne, is at the center of all of these universes. In this model Ellen White’s statement that the saints will one day “gaze upon the glory of creation,—suns and stars and systems, all in their appointed order circling the throne of Deity” [15, p677] makes a lot of sense.

You may be tempted to dismiss all I have written here as idle speculation. We who live in the end of the twentieth century in a highly materialistic culture are tempted to be passive observers, rather than allowing our minds to get in and wrestle with the greater facts that are not always visible to the naked eye. Robert Boyle lamented, “Sad to say our culture and even our academic structures seem to value only the shallow conceptual word, the useful, limited, rational word, the ‘scientific’ word available to the eye.” [4]

During the Mau Mau war of independence in Kenya, all of the Kikuyu people had to take an induction oath into the Mau Mau or flee from certain death. The oath involved drinking blood as a sacrifice to the animist spirits and certain immoral acts. I’ve heard Mrs Wangai tell her story of how she escaped taking the oath. As a teenager, she was brought before the council who demanded she take the oath. She refused and so was forced into the interior of a windowless hut to hopefully reconsider or await execution. Her captors stood guard at the only door while she lay bound on the floor, praying. Suddenly she had the sensation that she was moving. Looking
towards her feet she noticed a hole in the wall through which her feet and body were slowly moving. Her bonds fell off and she found herself outside the rear of the hut. After standing dazed and unbelieving for a few moments she fled into the deep bush and escaped. But not before looking back at the hut and seeing no hole in the wall. Sometime later she returned with others to the hut and there was no sign that there had ever been a hole in the wall.

One way to explain this, and Christ’s sudden appearance in the securely locked upper room is simply to say that it was a miracle. And who would question that they were both miracles, unless they refuse to accept that they happened? If some being had the power to move in a fourth spatial direction, they could easily appear in a locked room without going through any doors or walls. They could easily move someone out of a hut, without passing through the wall. To the person whose senses are restricted to three dimensions, there would appear to be a hole through which they were moving. Furthermore, a little reflection will convince you that no lock or knot or chain can remain locked or tied or linked in four dimensions. So bonds would fall off as a natural consequence of moving in a fourth dimension.

Explaining these miracles in this fashion in no way minimizes the miraculous nature of the event. After all we have no ability to perceive or use a fourth dimension. But it does help us believe that our guardian angel can indeed be very close to us and yet remain unperceived. Heaven can, in this way, be very close to us and ready to come to our aid whenever we need them. Thus angels, good and evil can be close to us. They can be very real, not ethereal beings. They can have great strength. They can reveal themselves simply by entering our dimension. They can change forms easily simply by how they enter our space. They can perform healings such as removing tumors without breaking the skin. As Paul said, “We wrestle not against flesh and blood” [2, Ephesians 6: 12]. Angels are very different creatures than we.

This concept I have just expressed is not new. As early as 1650 Henry More wrote that ghosts inhabited 4 dimensions and that we only saw them when they decided to enter our specific 3-dimensional subspace.
Incorporation into Curriculum and Instruction

There are several natural places to bring implications of infinity and higher dimensions into one's teaching. The calculus sequence is one such place.

During the presentation of limits, one uses the concept of a bounded infinity. I usually talk about the countability of the rational numbers and the uncountability of the irrational numbers very briefly. I also like to talk about the infinitesimals very briefly at this point. God's ability to handle affairs of the universe in a small amount of time (real time) can be brought in here. These topics are presented as asides and the students can take them or leave them.

During the discussion of improper integrals and also during the discussion of infinite sequences and series, one talks about unbounded infinities. Most people intuitively think about infinity as being in the same ball park as the national debt. The Christian doctrine that God is infinite can be mentioned here as the overwhelming reality of the magnitude of infinity begins to be grasped by the student.

In higher level math classes these topics can be reintroduced and expanded on as time allows. Linear algebra is an easy place to talk about the fourth dimension. Any class where Cantor set theory is discussed is also a natural. After all, that is where this whole paper got started!

Conclusion

Where will all this lead us? Can anything so abstract as infinity ever find "practical" applications in the real world? No one can really tell.

The history of mathematics is replete with discoveries which at first seemed to be totally abstract, but which later turned out to be of the utmost value to other sciences. We have seen how non-Euclidean geometry was at first accepted as a purely theoretical creation, but later found its way into the general theory of relativity. An even more dramatic example is that of group theory, a branch of algebra which only a century ago was regarded as the most abstract of all mathematical creations, and which today has become an indispensable tool in almost every branch of physics. These examples show that the course of mathematics, as of any science, is quite unpredictable, and one should never dismiss the possibility that some obscure branch of it may suddenly come to the forefront. As the physicist Niels Bohr (1885-1962) once said, "It is very hard to predict—especially the future!" [9, p 256]
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